**Khachaturyan Eigenstrain Method**

Having derived the elastic energy and the mechanical equilibrium equation,

(1)

There are three independent PDEs that we must solve (). We can now proceed to solve the partial differential equation (PDE) for the displacement fields. Khachaturyan’s method is an elegant solution for periodic boundary conditions (PBC).

Fourier transforming (the derivative operator become multiplication with ) both sides gives:

Let us define the inverse Green’s elastic tensor as:

(4)

Green’s elastic tensor is a second rank tensor, a matrix. Green’s elastic tensor is the inverse of the tensor defined in Equ 4 and is a Fourier space-varying tensor:

The inverse Fourier transform of the Fourier-space Green’s elastic tensor gives a Green’s function solution for the mechanical equilibrium condition [PH Dederichs and G. Leibfried, Elastic Green’s Function for Anisotropic Cubic Crystals]. Only special case of the elastic tensor give an analytical Green’s function solution.

Note the symmetry, because The PDE now simplifies to:

The strain in Fourier space can be found as:

(9)

(10)

(11)

(12)

Later we will show that: . The heterogeneous strain is:

(13)

**Simplification of the Heterogeneous Relaxation Energy**

We can analytically express the heterogeneous relaxation energy in terms of the eigenstress and Green’s elastic tensor.

(14)

We use the integral multiplication theorem to simplify the first term of Equ 14:

(15)

(15)

(15)

We use the following symmetries:

(16)

Matrix multiplication in Einstein notation is given by, where andare matrices,

(17)

Therefore,

(18)

(19)

The second term of Equ 14 is:

(20)

The heterogeneous strains are:

(21)

We use Equ 19 to simplify the following:

Using the following,

We arrive at:

Therefore, we have:

The above derivation agrees with literature [Yu U Wang, Yongmei M. Jin, AG Khachaturyan, Phase field microelasticity theory and modeling of elastically and structurally inhomogeneous solid]. The only difference between AG Khachaturyan and our derivation is their definition of the Fourier Transform. They define . This leads to their factor of versus our factor.